A NONSTATIONARY NOCTURNAL DRAINAGE FLOW MODEL

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Abstract. The evolution and structure of the steady state of an idealized nocturnal drainage flow over a large uniformly-sloping surface are studied using a nonstationary model with a height-dependent eddy diffusivity profile and a specified surface cooling rate. The predicted mean velocity and temperature profiles are compared with Prandtl's stationary analytical solutions based on the assumption of a constant eddy diffusivity in the drainage layer. The effects of important physical parameters, such as the slope angle, surface cooling, atmospheric stability, and surface roughness, on the steady drainage flow are investigated.

1. Introduction

The nocturnal drainage flow considered in this paper is a gravitationally induced local wind on a slope caused by the radiative cooling of the surface. This well-known phenomenon, usually referred to as the katabatic or downslope wind, may also occur in the daytime over ice- or snow-covered slopes. The local winds have a perceptible effect on climate, crops, and human welfare, especially if a source of pollution is involved. Hence, an understanding of the physical processes and structure of these flows is important.

Several analytical and numerical model studies of the drainage flow have been reported in the literature. Summaries of this work and related references can be found in Defant (1951), Thyer (1966), Gutman (1972), and Manins and Sawford (1979). In a pioneering theory of the steady slope wind over a large homogeneous sloping plane, Prandtl (1942) derived simple analytical expressions for the mean wind speed and potential temperature profiles as functions of the distance normal to the slope, surface temperature deviation from a reference state value, and a scale height of the flow. He assumed the flow to be stationary, the eddy diffusivities of momentum and heat (K_m and K_h) to be constant and equal, and the slope angle to be small. Defant (1949) extended Prandtl's theory to include the oscillatory nature of the slope wind and large slope angles and, using $K_h/K_m = 1.4$, obtained good agreement between predicted and observed wind profiles on a steep slope in the Innsbruck range. Lettau (1966) also found a satisfactory fit between Prandtl's theory and the observed wind and temperature distributions in a katabatic flow on the south polar plateau.

Nevertheless, the height dependence of the eddy diffusivities was not taken into account in Prandtl's theory, and quantitative comparison then reduced to a determination of a constant K coefficient for an arbitrarily estimated value of the surface

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temperature deviation. This paper describes a nonstationary nocturnal drainage flow model using a variable eddy diffusivity profile based on considerations of the flow structure, surface cooling rate, ambient atmospheric conditions, and surface characteristics. Numerical results on the evolution and structure of the steady drainage flow are presented. The effects of important physical parameters, such as the slope angle, surface cooling, atmospheric stability, and surface roughness, on the drainage flow are investigated.

2. The Model

We consider the evolution of an idealized drainage flow over a large homogeneous plane surface, sloping uniformly at an angle β to the horizontal (x, y) plane. With the origin 0 located somewhere on the surface, we adopt a slope-aligned rectangular coordinate system (s, y, n) such that the s axis is oriented along the fall-line vector, y axis in the lateral direction, and n axis normal to the surface (Figure 1). Because of

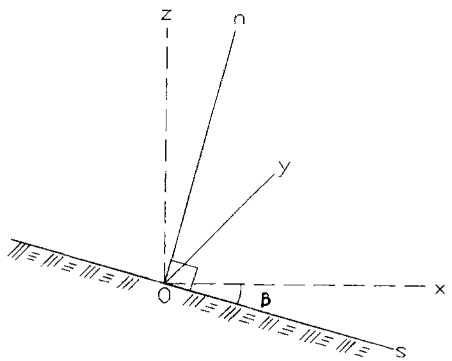

Fig. 1. Schematic diagram of the slope-aligned coordinate system.

the homogeneity of surface roughness and temperature, and the flow symmetry in this large plane problem, all components of motion are assumed to depend only on the normal height above the surface (n) and time (t). The Coriolis effects are negligible except for very small slope angles. Then the governing equations for the katabatic flow in the absence of an external wind can be written (Rao and Snodgrass, 1979) as

$$\begin{aligned} \partial U/\partial t &= -(g/\theta_0)\theta' \sin\beta + \partial(K_m\, \partial U/\partial n)/\partial n \\ \partial\theta/\partial t &= U\gamma \sin\beta + \partial(K_h\, \partial\theta/\partial n)/\partial n\,. \end{aligned} \tag{1}$$

Here, U is the mean flow velocity parallel to the s axis, $\theta'(n) = \theta - \theta_0$ is the deviation of the mean potential temperature θ from an undisturbed atmospheric reference state distribution $\theta_0(z)$, defined such that $\partial\theta_0/\partial t = 0$ and $\partial\theta_0/\partial z = \gamma = \text{constant} > 0$, and g is the acceleration due to gravity. The first equation states that the downslope acceleration is proportional to the imbalance between the drainage and turbulent friction forces. The second equation indicates that the cooling rate of the drainage layer is equal to the imbalance between the turbulent heat flux divergence and rate of downslope-warming due to the stable stratification of the ambient atmosphere.

For the flow over a sloping plane with β not less than several degrees and a characteristic value of the surface temperature deviation of at least a few degrees in magnitude, the gradients of mean temperature deviations normal to the slope are typically an order of magnitude larger than the temperature gradients in the undisturbed atmosphere and, hence, the latter can be neglected (Gutman, 1972). This simplifies the mean temperature equation to

$$\partial\theta'/\partial t = U\gamma \sin\beta + \partial(K_h\, \partial\theta'/\partial n)/\partial n\,. \tag{2}$$

The eddy diffusivities of momentum and heat are calculated as follows:

$$K_{m,h} = l_{m,h}(aE)^{1/2}\,, \qquad l_{m,h} = kn/\phi_{m,h} \tag{3a}$$

where $l_{m,h}$ are the turbulent mixing lengths, E is the turbulent kinetic energy (TKE), $k = 0.35$ is von Kármán's constant, and $a = 0.2$ is a proportionality constant (Delage, 1974; Nieuwstadt and Driedonks, 1979). The nondimensional wind shear (ϕ_m) and potential temperature gradient (ϕ_h) are specified by the expressions due to Businger *et al.* (1971) for the stable case; E is calculated from the TKE equation,

$$\begin{aligned}\partial E/\partial t = {} & \partial(K_m\, \partial E/\partial n)/\partial n + K_m(\partial U/\partial n)^2 - (g/\theta_0)K_h(\partial\theta'/\partial n + \gamma\cos\beta) - \\ & - b(aE)^2/K_m\,,\end{aligned} \tag{3b}$$

where $b = 1 - R_f$, and R_f is the local flux Richardson number computed directly from the model such that $0 \leqslant R_f \leqslant R_{fcr} = 0.21$. Numerical solutions of the closed equation set (1) to (3), subject to the boundary and initial conditions given below, were obtained using a Dufort–Frankel explicit finite-difference scheme. A logarithmic transformation of the vertical coordinate was used to obtain a fine computational mesh near the ground.

At $n = z_0$, the aerodynamic surface roughness, the boundary conditions are

$$U = 0\,, \qquad \theta' = -\Delta_0\,, \qquad \partial E/\partial n = 0\,, \tag{4a}$$

where $\Delta_0(t)$ is the magnitude of the surface temperature deviation, defined such that $\Delta_0(0) = 0$. The flux-profile relations of Businger *et al.* (1971) for the stable case are used to calculate the turbulent fluxes in terms of mean field variables at the lower boundary. At $n = H > h$, where $h(t)$ is the thickness of the drainage layer, not known *a priori* but determined from the solution of the problem, we specify

$$U = \theta' = E = 0\,. \tag{4b}$$

For a pure katabatic flow in the absence of an external wind, Equation (4b) also describes the conditions at $t = 0$ since the air is initially at rest. For $t > 0$, the air near the surface cools due to the rapid radiative heat loss from the surface on a clear night.

A flow develops when the cooled air layer is heavy enough to overcome friction and slide down the slope. Presumably this surface drainage flow will be laminar in the early stage of its development, but a transition to turbulence occurs within a short time depending on the flow Reynolds number. As the surface continues to cool, the wind speed and the turbulent mixing increase, and the mean quantities of the atmospheric drainage flow are determined from Equation (1) in which the molecular contributions to the momentum and heat fluxes are neglected. Even if the molecular terms were included in the formulations, the model integration cannot be started from rest due to numerical instabilities encountered in the transition from the laminar to the turbulent regime.

In order to obtain a numerical solution of the turbulent drainage flow described by Equations (1) to (4), therefore, the initial conditions on U, θ', and E must be given at a time $t = t_0 > 0$. These initial distributions may be approximated, in principle, by any compatible set of profiles that satisfy the boundary conditions (4a) and (4b), as well as the flux-profile relations at the lower boundary. Due to their simplicity and convenience, Prandtl's analytical solutions (see Section 3.2) were used to specify the approximate initial profiles following a procedure discussed by Rao and Snodgrass (1979). The governing equations were then numerically integrated in time, holding the boundary conditions constant, until the equilibrium distributions consistent with our height-dependent eddy diffusivity formulations evolved; these distributions were then used as the correct initial conditions at $t = t_0$. This procedure ensures that the model results, presented below, are independent of the approximate initial conditions specified to start the numerical integration.

3. Results and Discussion

3.1. Evolution and Structure of Steady State Flow

The evolution of the mean velocity and potential temperature profiles, and the corresponding eddy diffusivity profiles, in a nocturnal drainage flow are shown in Figures 2(a), (b), and (c), respectively. The flow parameters used in the computations are $\beta = 10°$, $\gamma = 0.006$ °C m^{-1}, $z_0 = 0.1$ m, and a surface cooling rate of 2 °C hr^{-1} for $0 < t < 1$ h. For $t \geqslant 1$ h, the surface temperature was kept constant such that $\Delta_0 =$ 2 °C. Since the surface temperature variation specified here was rather arbitrary, the time t shown in the figures should be interpreted only as an indicator of the corresponding surface temperature-deficit (Δ_0), and not the physical time *per se*. A striking feature of the slope wind profile is the occurrence of a prominent velocity maximum (U_{max}) near the surface. The magnitude and location of this low-level drainage jet vary as a function of $\Delta_0(t)$, among other factors, as shown in Figure 2(a). The small positive deviations at the top of the temperature profiles (Figure 2b) can be explained as follows. Due to turbulent friction, the cooled sinking air parcels also drag down the adjacent non-cooled air layers. During their descent, the latter become warmer than the neighboring layers which are not set into motion. These

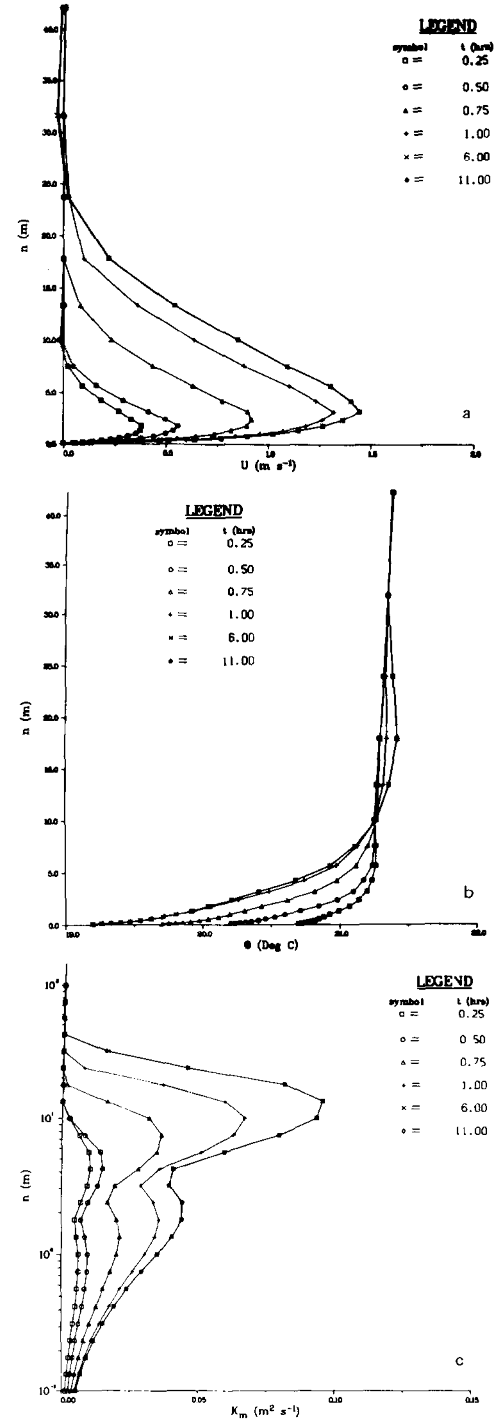

Fig. 2. Predicted evolution of (a) mean velocity, (b) mean potential temperature, and (c) eddy diffusivity of momentum profiles in the drainage flow. The profiles for $t = 6$ and 11 h, which coincide exactly, indicate the establishment of a steady state. See text for details and flow parameters.

layers, upon being warmed by the turbulent transfer of heat, start to ascend and form a weak counterflow at the top of the drainage layer (Figure 2a).

A stationary solution of the drainage flow described by Equations (1) to (4) can be obtained when Δ_0 approaches a constant value. In this example, with Δ_0 held constant at 2 °C for $t \geqslant 1$ h, a steady state, with a peak velocity at 1.5 m s^{-1}, was attained in a few hours as indicated by the invariancy of profiles at $t = 6$ and 11 h which coincide exactly in Figure 2.

The steady-state profiles of the turbulent kinetic energy E, shear stress $\overline{uw} = -K_m\, \partial U/\partial n$, and heat flux $\overline{w\theta} = -K_h\, \partial\theta'/\partial n$ are shown in Figures 3(a), (b), and (c), respectively. The kink in the TKE profile corresponds to the maximum in the velocity profile, where the local shear production is zero. At this level, the non-zero energy is

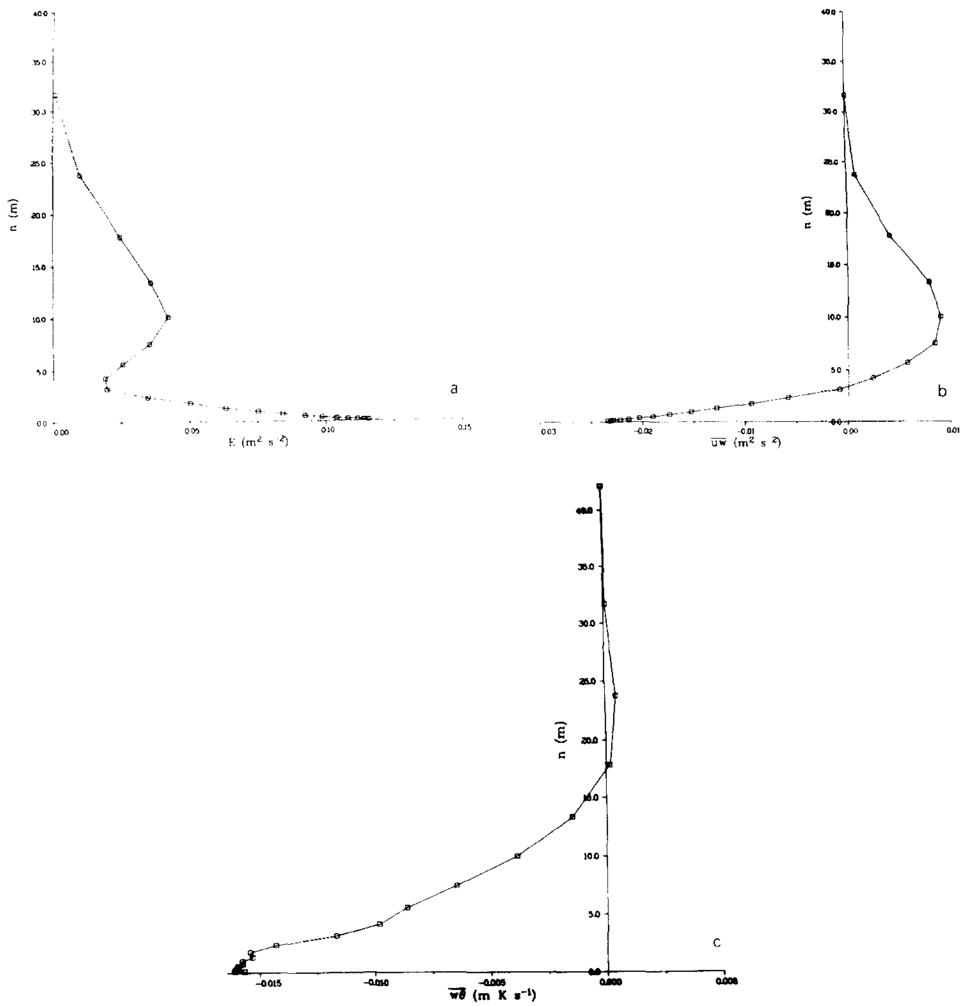

Fig. 3. Steady state profiles of the turbulent (a) kinetic energy, (b) shear stress, and (c) heat flux in the drainage flow ($\beta = 10°$, $\Delta_0 = 2$ °C, $\gamma = 0.006$ °C m^{-1}, $z_0 = 0.1$ m).

only due to the turbulent transport from the surface. The shear stress (Figure 3b) is negative near the ground, goes through zero at the U_{max}-level, and remains positive in the outer layer. A positive maximum in $\overline{uw}$ occurs at a level corresponding to the inflection point in the velocity profile where $\partial^2 U/\partial n^2 = 0$. The heat flux (Figure 3c) is negative in the drainage flow with a maximum near the surface. The slight positive heat flux at the top is due to the positive temperature deviations discussed above. Based on the turbulence profiles, the thickness h of the stationary drainage layer in this problem is 32 m.

3.2. Comparison with Prandtl's theory

Prandtl's analytical expressions (see Defant, 1951) for the mean wind and temperature deviation profiles in the stationary slope wind can be written as

$$U(\xi) = U_c e^{-\xi} \sin \xi , \qquad \theta'(\xi) = -\Delta_0 e^{-\xi} \cos \xi , \tag{5a}$$

where

$$\xi = (n - z_0)/l ,$$
$$l = (2K_m/N \sin \beta)^{1/2} \alpha_k^{1/4} , \qquad U_c = (\Delta_0 N/\gamma)\alpha_k^{1/2} . \tag{5b}$$

In the above, K_m is a constant eddy diffusivity, $\alpha_k = K_h/K_m$ is a constant of order unity, $N = (g\gamma/\theta_0)^{1/2}$ is the Brunt-Väisälä frequency in the stable ambient atmosphere, and l and U_c, respectively, are the characteristic height and velocity of the drainage flow. The original Prandtl solutions, which do not explicitly depend on the surface roughness parameter, are slightly modified here, by including z_0 in the definition of ξ in Equation (5b), to be consistent with our formulations. Equations (5) indicate that the slope wind is effectively confined to the layer $z_0 \leqslant n \leqslant \pi l$, and it reaches its first and absolute maximum, $U_{max} = 0.3224\ U_c$, at a normal height $n_{max} = \pi l/4 + z_0$ where $\partial U/\partial n = 0$. Therefore, U_c and l, and hence the mean wind and temperature distributions from Equations (5), can be determined if U_{max} and n_{max} are known from a field or a numerical experiment.

Figures 4(a), (b), and (c) show the comparison of U, θ', and K_m profiles, respectively, between the present model calculations and Prandtl's theory. The latter, using $\alpha_k = 1.35$, underpredicts the drainage layer thickness and yields smaller velocities at all levels, while slightly overpredicting θ' in the surface layer. However, the general profile shape and behavior are similar in both models. This suggests that Equations (5) can be used to provide a fair approximation to the velocity and temperature profiles in a katabatic flow if the peak jet velocity and its height are independently determined from field observations. Earlier work by Defant (1949) and Lettau (1966) support this conclusion.

3.3. Sensitivity analyses

In order to study the relative effects of important physical parameters on the steady nocturnal drainage flow, each parameter was varied while keeping the others

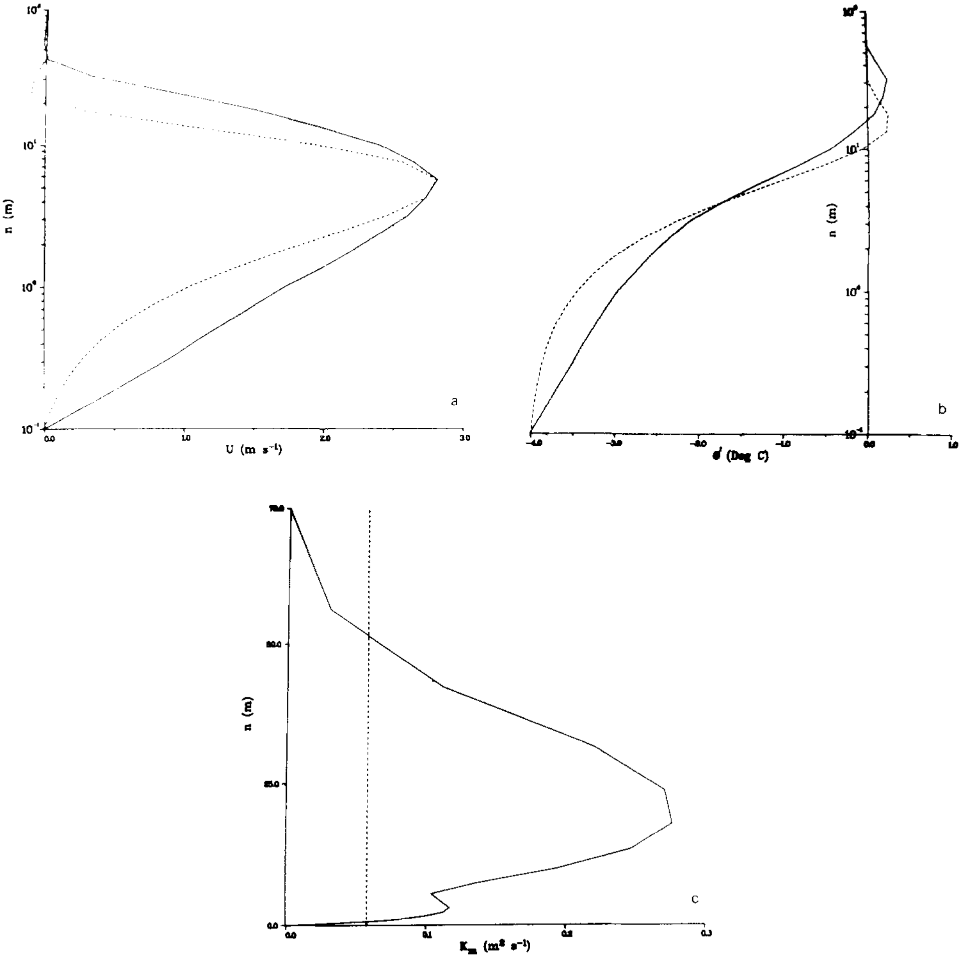

Fig. 4. Comparison of the steady-state (a) mean velocity, (b) mean potential temperature deviation, and (c) eddy diffusivity of momentum profiles predicted by the present model (——) and Prandtl's theory (-----).

constant in the computations. The parameters of the control run are $\beta = 10°$, $\Delta_0 = 4$ °C, $\gamma = 0.006$ °C m^{-1}, and $z_0 = 0.1$ m. Figures 5(a) and (b), respectively, show the steady-state profiles of U and θ' for three different slope angles. As β increases, the drainage velocity at a given height increases near the surface and decreases in the outer layer. However, the maximum velocity (U_{max}) is not very sensitive to the slope angle; the physical explanation is that while it is more difficult for the air to sink along a steeper slope, the drainage force, which is proportional to sin β, is also larger. As the slope increases, comparatively less cooling (Figure 5b) of the air layer is required to produce the same drainage force. It can be seen that the drainage layer becomes shallower and n_{max} decreases as the slope increases.

Figures 6(a) and (b) show the steady-state U and θ' profiles for three different values of the surface temperature-deficit (Δ_0). As the latter increases, U and θ'

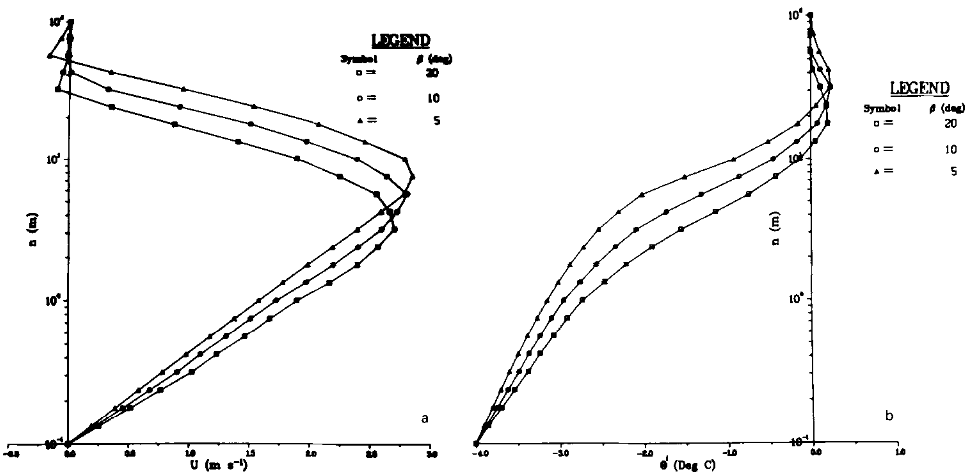

Fig. 5. Effects of the variation of the slope angle β on the steady-state (a) U, and (b) θ' profiles in the drainage flow ($\Delta_0 = 4$ °C, $\gamma = 0.006$ °C m^{-1}, $z_0 = 0.1$ m).

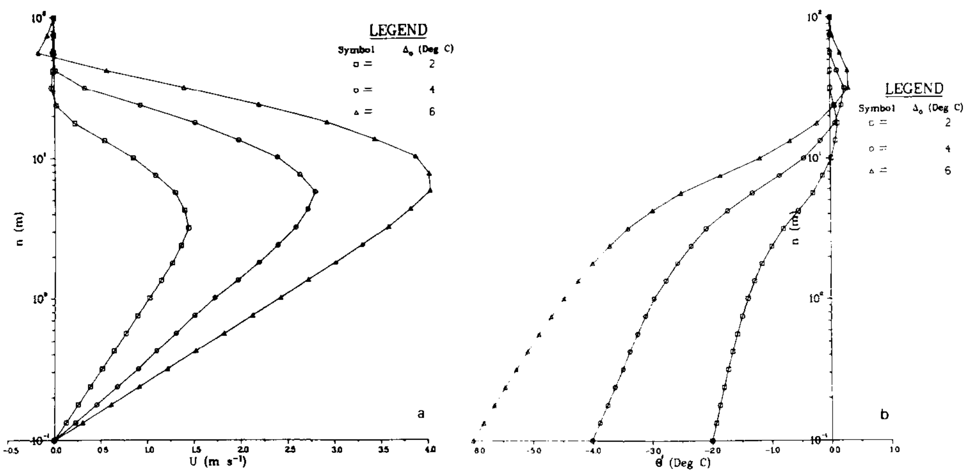

Fig. 6. Effects of the variation of the surface temperature-deficit Δ_0 on the steady-state (a) U, and (b) θ' profiles in the drainage flow ($\beta = 10°$, $\gamma = 0.006$ °C m^{-1}, $z_0 = 0.1$ m).

increase at all levels; $U_{\max}$, $n_{\max}$, and h also increase. The variation of the maximum drainage velocity is directly proportional to the surface cooling. The effects of atmospheric stability on U and θ' profiles in the steady drainage flow were also investigated (not shown) using three different values of γ between adiabatic and isothermal conditions. The results indicate that the more stable the ambient atmosphere, the less are the velocity and cooling in the drainage flow; the boundary-layer thickness also is smaller.

The effects of the surface roughness on the equilibrium profiles of U, θ', and K_m in the drainage flow are shown in Figures 7(a), (b), and (c). As z_0 decreases, the turbulent friction force decreases; however, the cooling in the layer and the drainage

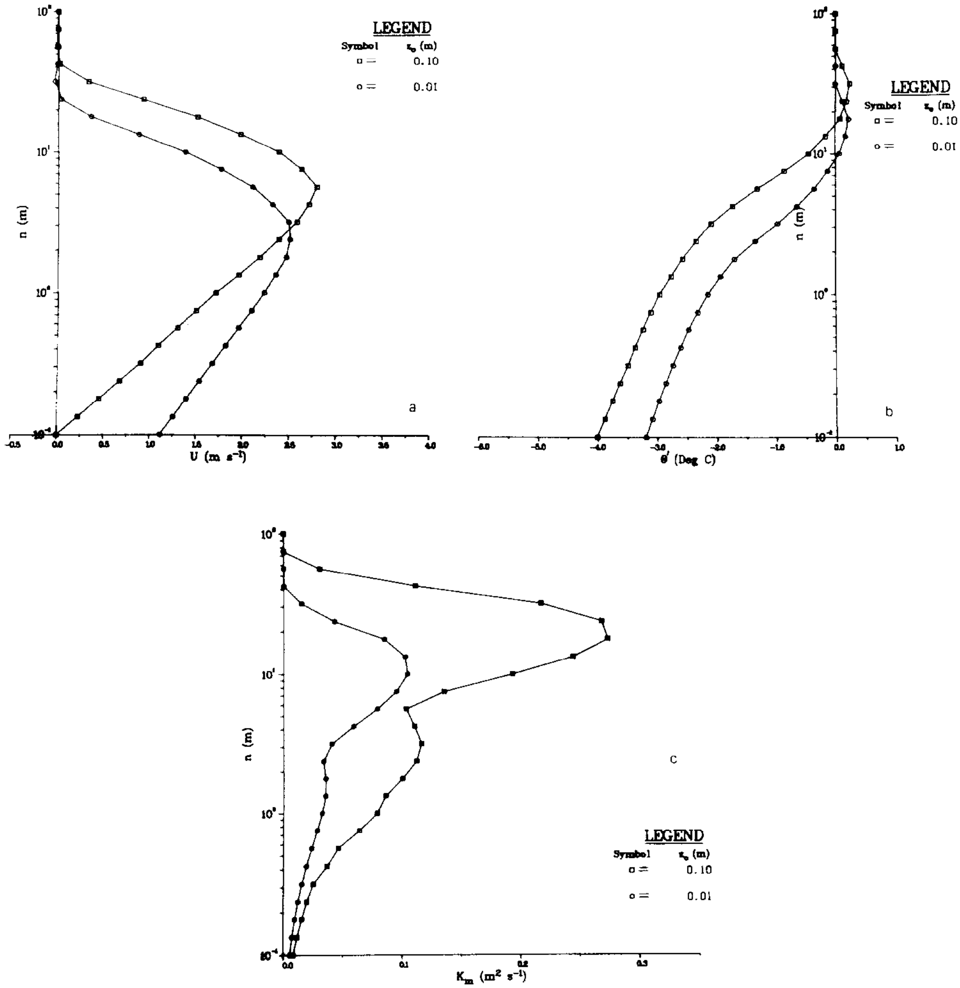

Fig. 7. Effects of the variation of the surface roughness parameter z_0 on the steady-state (a) U, (b) θ', and (c) K_m profiles in the drainage flow ($\beta = 10°$, $\Delta_0 = 4$ °C, $\gamma = 0.006$ °C m^{-1}).

force also decrease since all other flow parameters including the surface cooling (Δ_0) were kept constant in this study. Thus, as z_0 decreases, $U_{\max}$, $n_{\max}$, and h decrease as shown.

4. Conclusions

The evolution of an idealized nocturnal drainage flow over a large homogeneous uniformly-sloping surface has been investigated by a nonstationary model using a height-dependent eddy diffusivity profile and a specified surface cooling rate. For a constant surface temperature-deficit, the flow approaches stationarity. Though it is unlikely that long periods of this steady state occur in reality when the ground temperature is determined by the surface energy budget (Brost and Wyngaard, 1978), the model is useful in understanding the equilibrium structure of the katabatic flow. The evolution of the drainage flow can be predicted if the surface temperature or heat flux variation, among other parameters, is determined from field experiments. Due to lack of suitable data, this numerical model has not yet been tested directly against observations. However, the satisfactory fit of Prandtl's theory to the field data reported by Defant (1949) and Lettau (1966), and the mean velocity and density profiles obtained in the laboratory simulations of Ellison and Turner (1959) indirectly support the model predictions.

The simplifying assumptions that led to this one-dimensional model do not allow for the explicit accounting of the effects of the interfacial entrainment of ambient air into the drainage flow that are known to be important (Ellison and Turner, 1959; Manins and Sawford, 1979) for slopes of limited extent. Nevertheless, this model should yield satisfactory results for large sections of mountain slopes (Gutman, 1972), away from the ridges and valleys.

This drainage flow model is applicable only for relatively calm or light synoptic wind conditions. The extension of the model to account for the ambient winds and Coriolis effects, the conditions for the existence of a steady state, and the stationary analytical solutions are discussed by Rao and Snodgrass (1979).

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